Mathematics Takes Shape

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Geometric sculpture can be a potent tool for communicating mathematical ideas in a visual and tactile manner. While all art should be enriching and thought provoking, mathematically based art displays additional internal resonance through underlying relationships which appeal to one’s sense of system and logic. It draws upon and celebrates the visual and structural modes of thinking which bind art with mathematics.

As a mathematician and constructive sculptor, I create works that follow in a centuries-long tradition of mathematically informed art. My motivation, in part, is that I try to convey a sense of what I call the geometric aesthetic. I have the artistic conviction that the patterns and relations found in the classical geometry of three-dimensional structures can form a solid foundation for art that is beautiful, personally affecting, and visually engaging.

I am always gratified when viewers of my work, though not mathematically trained, are led to ask me questions of a mathematical character. Some are of a technical nature, e.g., How do you compute the correct angle for those parts to meet exactly? Others reach more to the heart of the matter, e.g., What is the pattern behind this? I am always happy to elucidate, as I feel that an artwork that evokes such questions has in some way been kissed by Euclid.

This photo-essay tries to convey these ideas directly—through images of my work. With just a brief caption for orientation, a successful sculpture should be able to speak for itself. For additional insight and mathematical descriptions of their underlying structure, see the detailed references given in my web pages, http://www.georgehart.com/.

Roads Untaken

Wood, 17-inch diameter. It is fabricated like a mosaic, with three colors of wood (yellowheart, paela, and padauk) and walnut "grout," over a hollow fiberglass sphere. I call the geometric form an "exploded propellerized truncated icosahedron." To plan it I designed a set of polyhedral operators to produce the topological structure, and a numerical relaxation method to proportion it with all edges tangent to a common sphere. Your eye can see it as light roads (connecting hexagons and pentagons) on a dark background, or as dark roads (connecting triangles) on a light background. Is there a light path from any hexagon to any other hexagon? Is there a dark path from any triangle to any other triangle?
Battered Moonlight

Papier-mâché over steel, 21 inches. This is a 6-edged, 1-sided surface (like a Möbius strip) with the rotational symmetries of an icosahedron (i.e., fifteen 2-fold, ten 3-fold, and six 5-fold rotation axes). To understand its form, imagine separating the twenty faces of an icosahedron, then reconnecting them with a half-twist ribbon replacing each of the thirty edges. Made of papier-mâché over a steel frame, its title comes from a poem by Elizabeth Bishop which came to mind after seeing the play of light over its painted, textured surface. Find a path for an ant to get to the other side without crossing an edge.
**Millenium Bookball**

Wood and bronze, 5 feet. Commissioned by the Northport Public Library, this sculpture was assembled at a community “barn raising.” There are 32 bronze tori connecting the books. The books which meet at any torus form a type of “propeller.” There are twenty 3-way propellers and twelve 5-way propellers. The spines of the books follow the edges of a rhombic triacontahedron. Look carefully to see that the tilt of the planes was chosen so each book is coplanar with another book partway around the other side.

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**Loopy**

Painted aluminum, eight feet tall. These thirty metal loops, each 10 feet long if unrolled, were drilled, formed into a curve, enameled, woven through each other, and fastened together with stainless steel bolts. Observe how the five colors are carefully arranged: each loop crosses one loop of the other four colors, each color appears once around each 5-sided opening, and the ends of each loop approach the ends of two other loops of the same color. The six loops of any one color are positioned as in the edges in a regular spherical tetrahedron.

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**I'd like to make one thing perfectly clear**

Acrylic plastic, 18 inches. Thirty identical pieces of acrylic plastic were heated in an oven to soften them and allowed to cool in a jig to set the appropriate shape. The edges were beveled to fit together in groups of five and the components were cemented together with jigging to hold their proper relative positions. As in many of my pieces, the work of making the jigs far exceeded the actual construction of the sculpture itself. Looking through any piece to the opposite side, the views of the sculpture seen through itself are distorted by refraction. Is it odd that a pure mathematical form corrupts itself?
Fat and Skinny

Wood and brass, 23 inches. Two spheroids, with very different “personalities,” are constructed from the same set of components (12 regular pentagons, 30 squares, 20 equilateral triangles, and 6 golden rhombi). They are arranged differently, as in a jigsaw puzzle with two solutions. In each case, the dark components (walnut pentagons and triangles) fit together as an icosidodecahedron. The light components (maple squares and rhombi) are arranged along three of the icosidodecahedron’s “equators” but there are two different ways to choose three equators.

Rainbow Bits

CD ROMs, 6 feet. Carlo Séquin commissioned this orb of 642 holographic CDs for the Computer Science building of the University of California at Berkeley. The form is based on the “propello-icosahedron,” with chains of CDs outlining the edges. I cut the slots in the CDs at my studio in New York, and shipped them to California in a box smaller than a one-foot cube. There I assembled the six-foot diameter structure in place, 20 feet up in the Soda Hall atrium where it hangs. Notice that the openings are equilateral triangles and symmetric “kites.”

Whoville

Aluminum, 35 inches. The form derives from an icosahedron and dodecahedron in mutually dual position, which would lie in the empty central region of the sculpture. The five-fold dimples correspond to the vertices of the icosahedron and the three-fold dimples (in the “basements” of the three-sided “buildings”) correspond to the vertices of the dodecahedron. The lines of the sculpture extend or parallel the edges of these implicit polyhedra. The rectangular form of each doorway is a golden rectangle and the triangles were chosen to create coplanar pairs.
Two 3-inch balls, 3D printing. Computer-generated models of novel polyhedra had their edges adapted into solid forms in two different ways. In the first, the edges remain lace-like at the surface of the sphere, while in the second, they are carried to the center. In both, the thickness of each edge is proportional to its length, which helps to make them seem more organic. 3D printing is currently expensive to produce. How many different path shapes are there in each?

Twisted Rivers, Knotted Sea

Steel, 7 feet tall. The geometric form underlying this four-foot diameter steel sculpture is based upon the stellation pattern of the icosahedron, but reorganized into thirty identical curved units. The steel pieces were cut with a computer-controlled laser cutter, folded to the proper angles, powder coated blue, woven together, and joined with stainless steel bolts. Ten pieces meet at each of the twelve bolted junctions. The twenty openings are three-way "whirlpools."